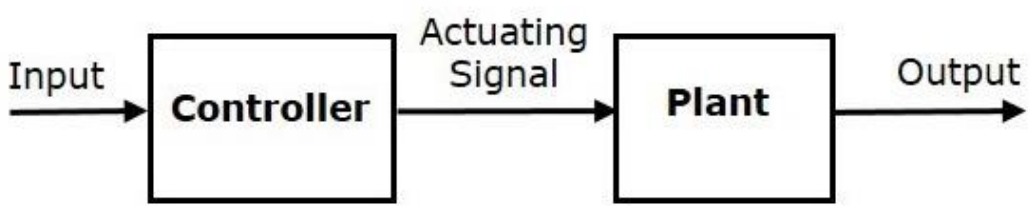
Lecture3: Block Diagram

1. Introduction to Block Diagram:

Block diagram is a pictorial representation of a control system showing inter-relation between the transfer function of various components. The block diagram is obtained after obtaining the differential equation and transfer function of all components of a control system.

Control Systems can be classified as **open loop control systems** and **closed loop control systems** based on the **feedback path**.

In **open loop control systems**, output is not fed-back to the input. So, the control action is independent of the desired output. The following figure shows the block diagram of the open loop control system.



Here, an input is applied to a controller and it produces an actuating signal or controlling signal. This signal is given as an input to a plant or process which is to be controlled. So, the plant produces an output, which is controlled.

An example of this is a traffic lights control system. Here, a sequence of input signal is applied to this control system and the output is one of the three lights that will be on for some duration of time. During this time, the other two lights will be off. Based on the traffic study at a particular junction, the on and off times of the lights can be determined. Accordingly, the input signal controls the output. So, the traffic lights control system operates on time basis.

In **closed loop control systems**, output is fed back to the input. So, the control action is dependent on the desired output. The following figure shows the block diagram of negative feedback closed loop control system.

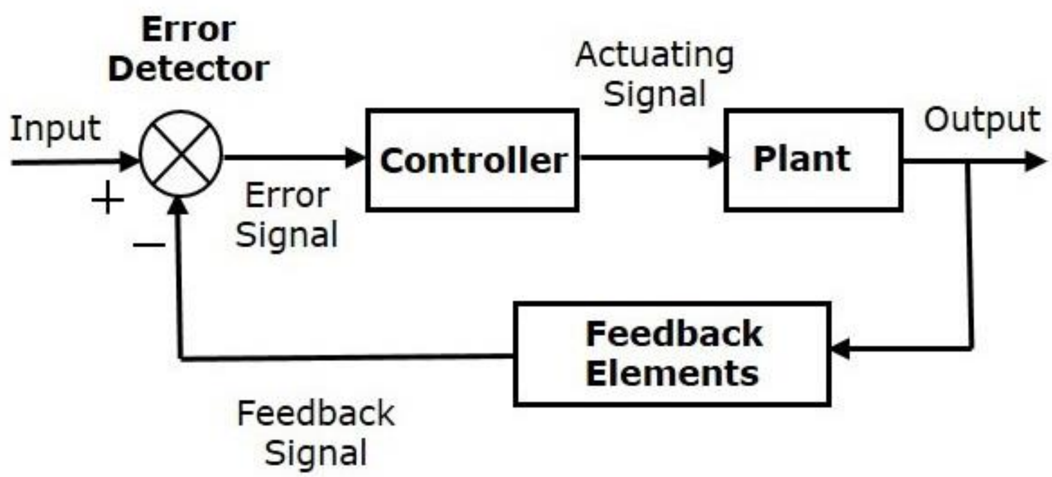


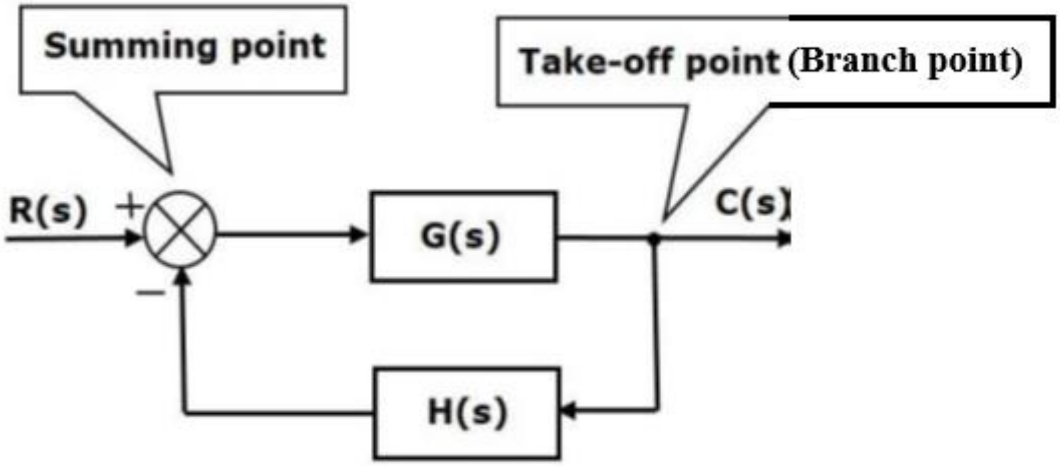
Figure (3.1): Block diagram components.

The error detector produces an error signal, which is the difference between the input and the feedback signal. This feedback signal is obtained from the block (feedback elements) by considering the output of the overall system as an input to this block. Instead of the direct input, the error signal is applied as an input to a controller. So, the controller produces an actuating signal which controls the plant. In this combination, the output of the control system is adjusted automatically till we get the desired response. Hence, the closed loop control systems are also called the automatic control systems.

Traffic lights control system having sensor at the input is an example of a closed loop control system.

2. Elements of Block Diagram:

The basic elements of a block diagram are a block, the summing point and the take-off point. Let us consider the block diagram of a closed loop control system as shown in the following figure to identify these elements.

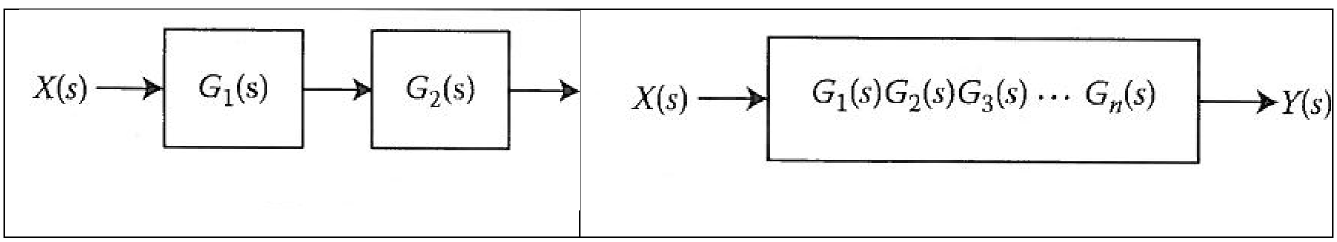


3. Block Diagram Reduction Rules:

For simplifying (reducing) the block diagram, which is having many blocks, summing points and take-off points, we follow **five rules** and repeat them till we get the simplified form, i.e., single block.

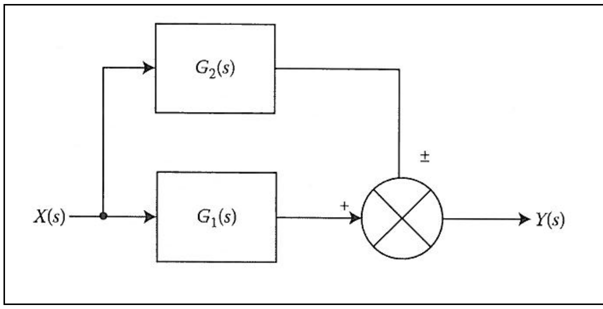
Note: The transfer function present in this single block is the transfer function of the overall block diagram.

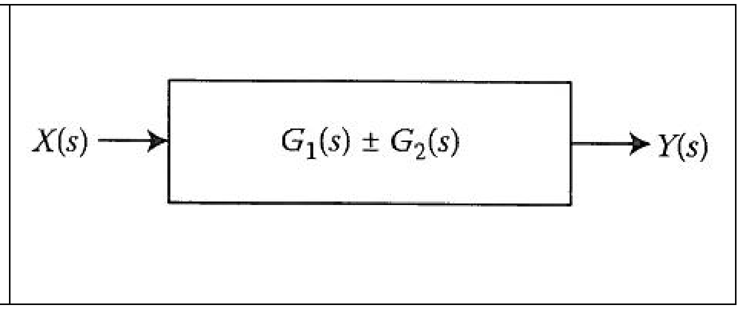
- **Rule 1:** Check for the blocks connected in series and simplify.



❖ Series connection is also called **cascade connection**. In the above figure, two blocks having transfer functions G1(s) and G2(s) are connected in series. The transfer function of the single block is the **product of the transfer functions** of those two blocks.

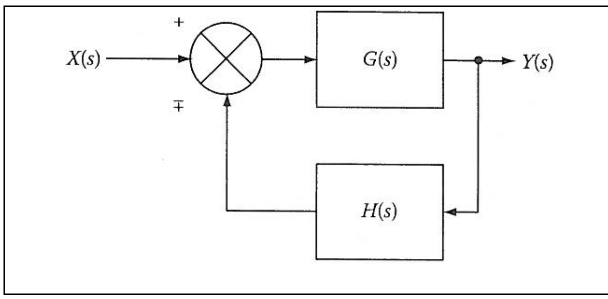
- **Rule 2:** Check for the blocks connected in parallel and simplify.

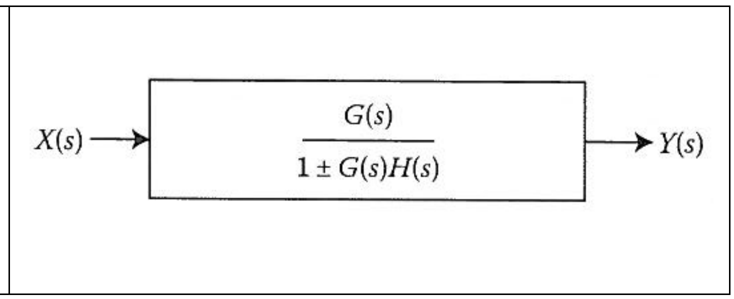




❖ The blocks which are connected in **parallel** will have the **same input**. In the following figure, two blocks having transfer functions G1(s) and G2(s) are connected in parallel. The outputs of these two blocks are connected to the **summing point**. The transfer function of the single block is the **sum of the transfer functions** of those two blocks.

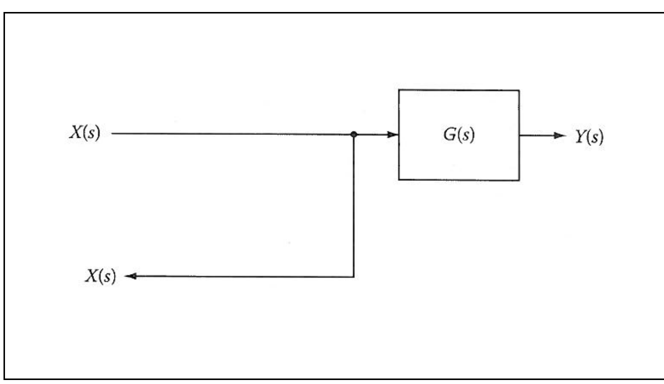
- **Rule 3:** Check for the blocks connected in feedback loop and simplify.

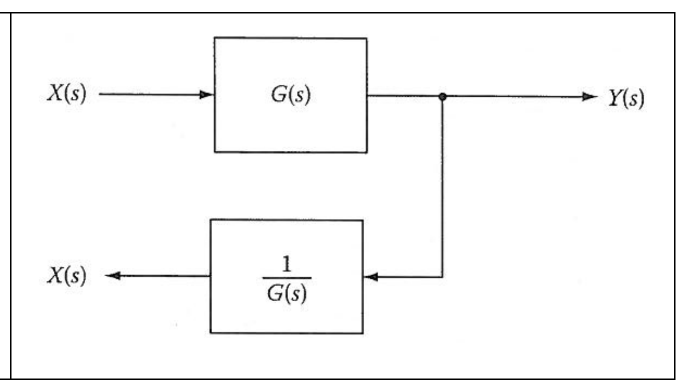




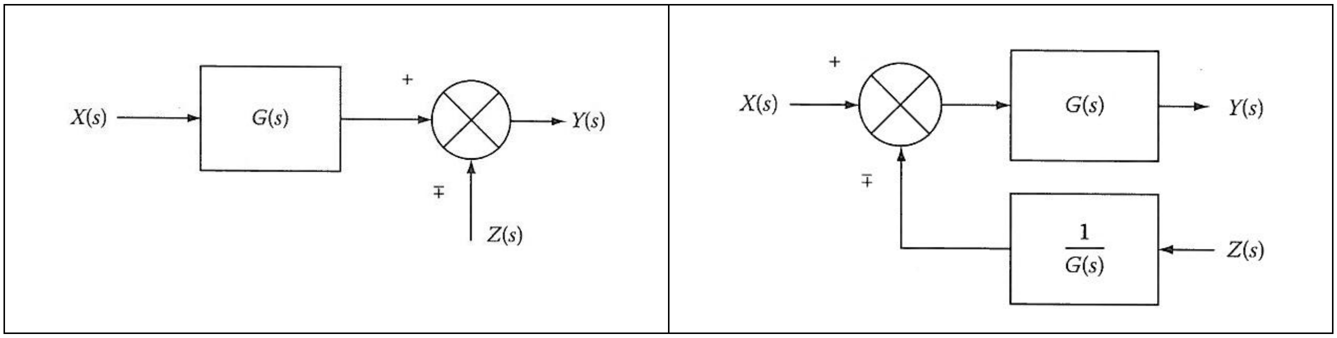
❖ There are two types of **feedback**: **positive feedback** and **negative feedback**. The transfer function of this single block is the closed loop transfer function of the negative/positive feedback.

- **Rule 4:** If there is difficulty with take-off point while simplifying, shift it towards right.

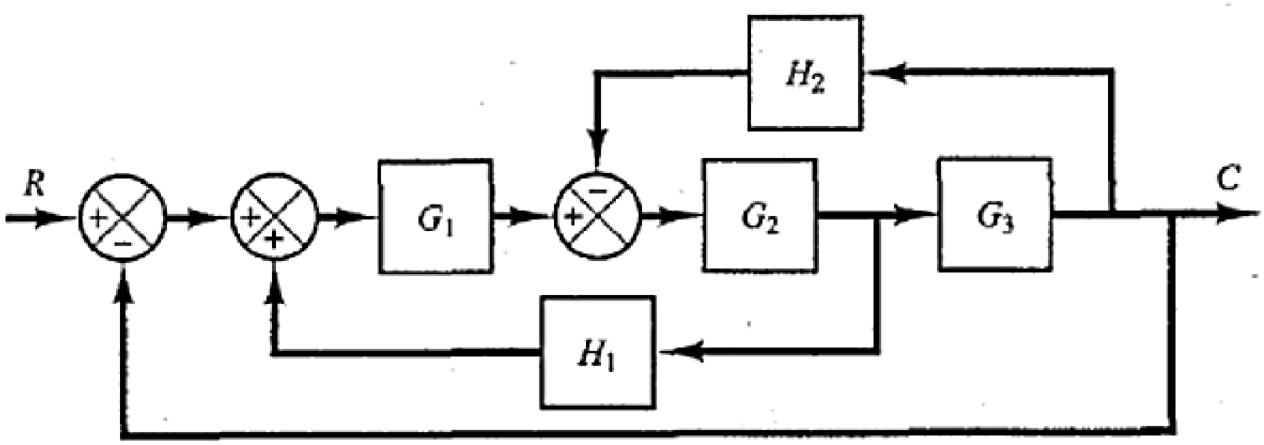




- **Rule 5:** If there is difficulty with summing point while simplifying, shift it towards left.

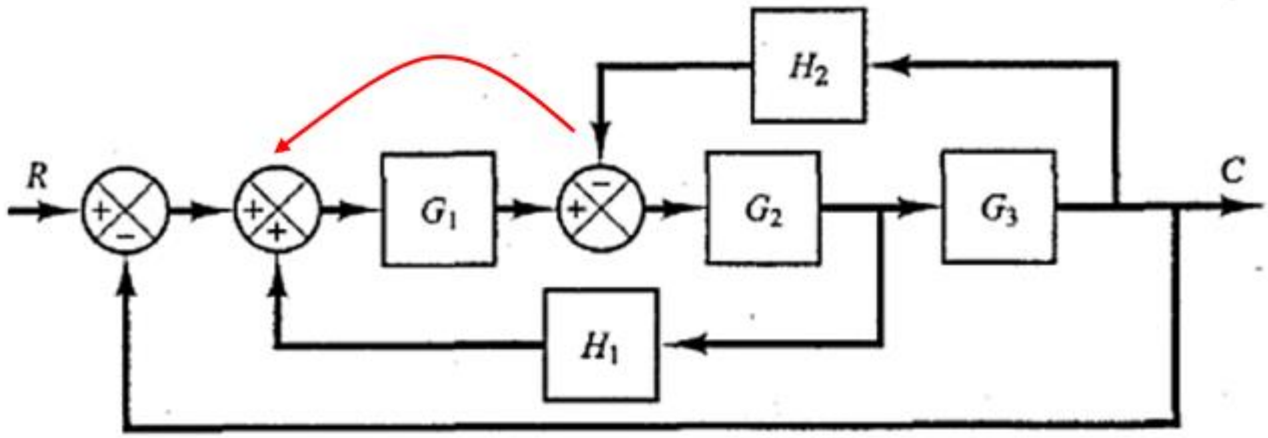


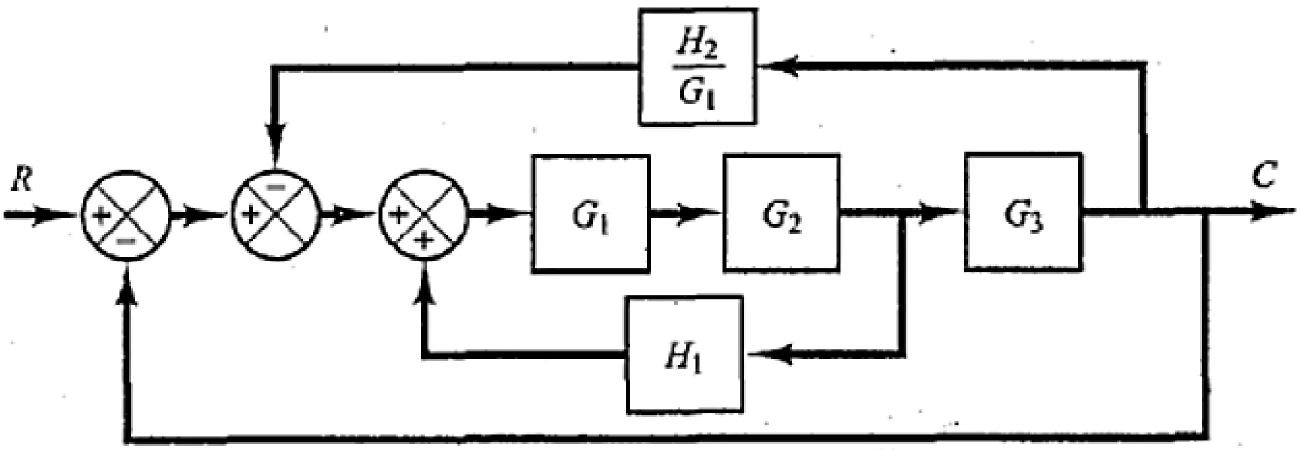
Example 1: Using the block diagram reduction technique, reduce the system shown in figure below to simplest possible form and find the transfer function.



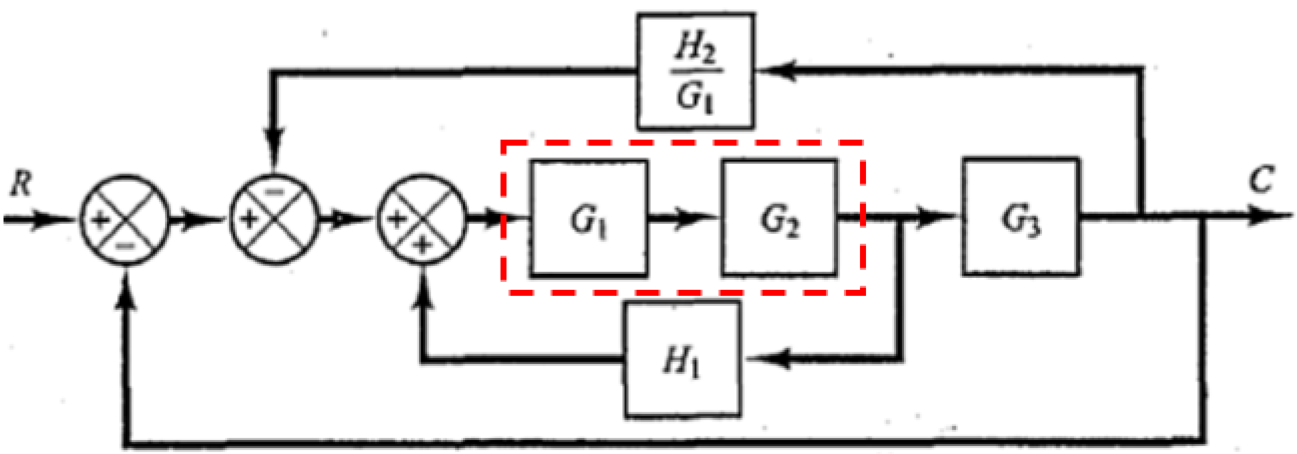
Solution:

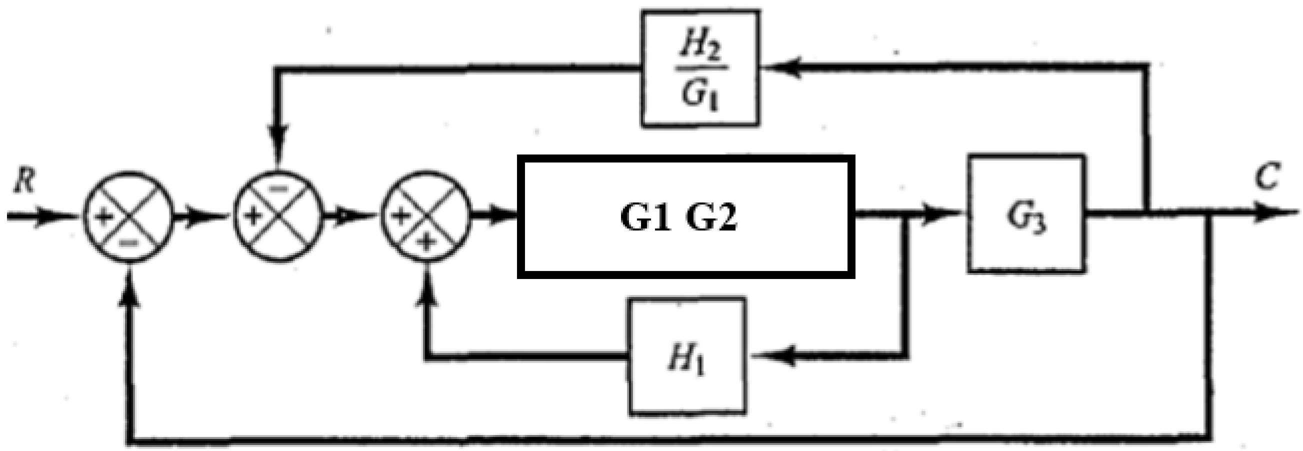
Step 1: There is difficulty with summing point after G_1 while simplifying. Thus, we use **Rule 5** to shift it towards left (before G_1).



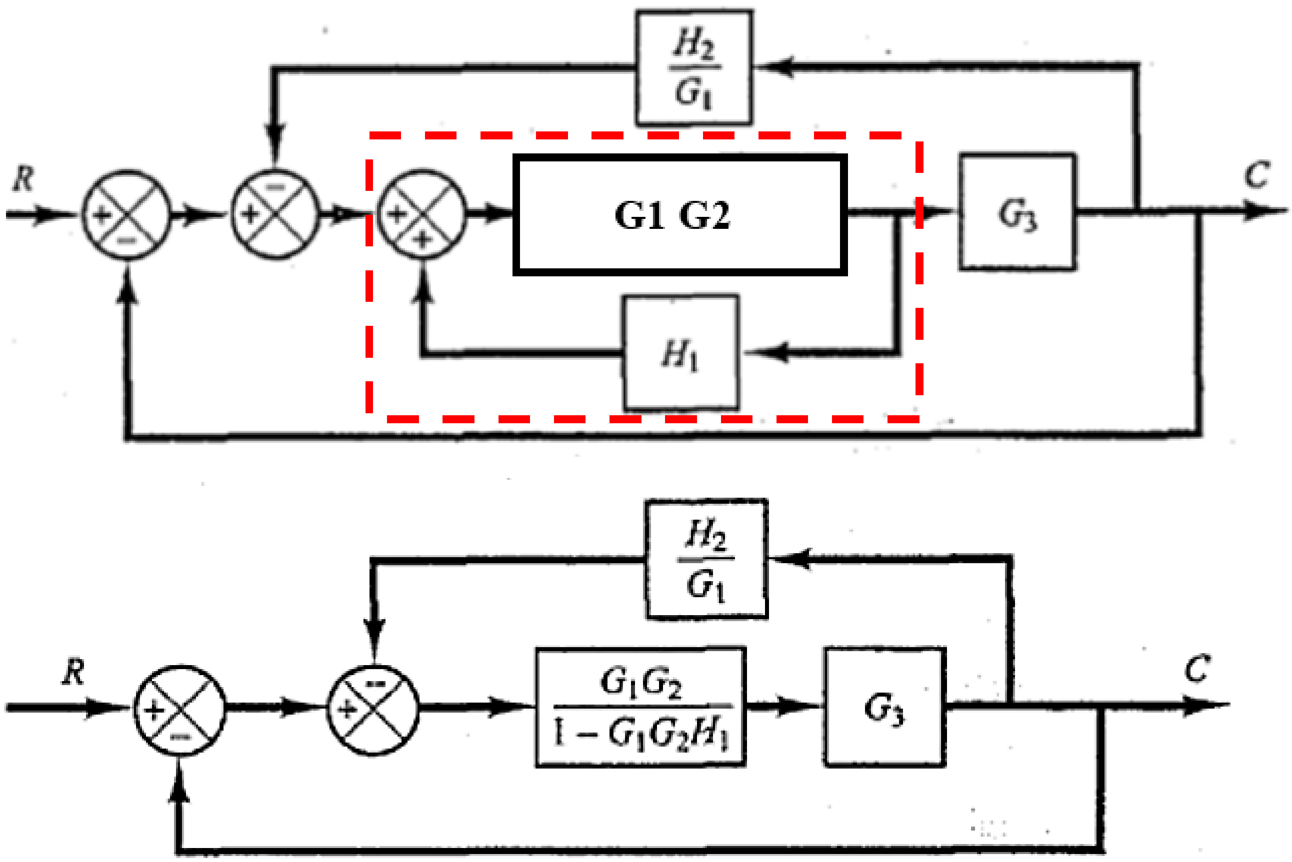


Step 2: Use **Rule 1** for blocks G_1 and G_2 which are connected in series to combine them into one block $G_1\ G_2$. The modified block diagram is shown in the following figure.

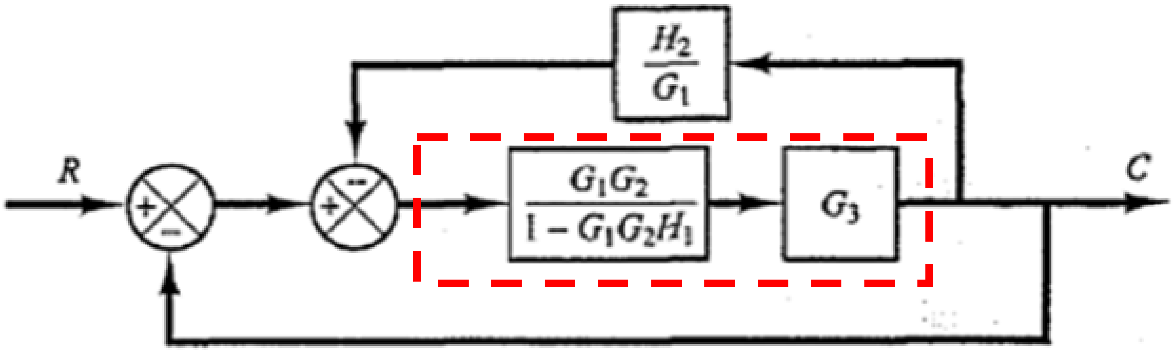


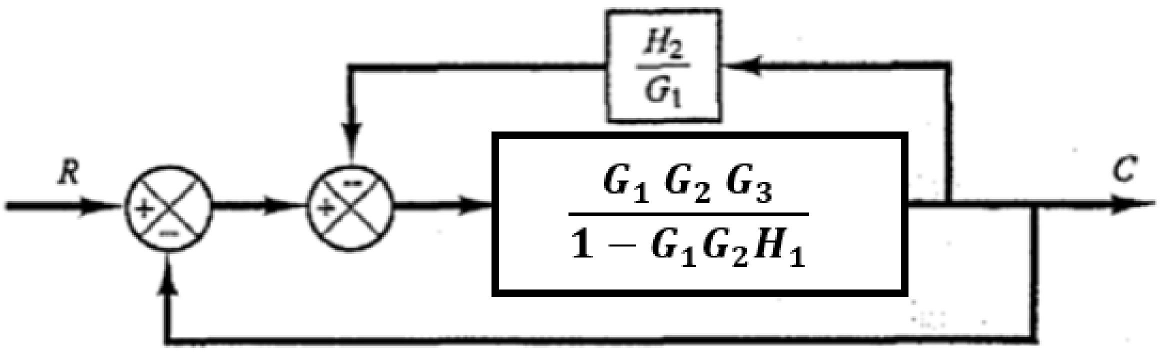


Step 3: Use **Rule 3** for blocks G_1G_2 and H_1 which are connected in positive feedback loop to combine them into one block. The modified block diagram is shown in the following figure.

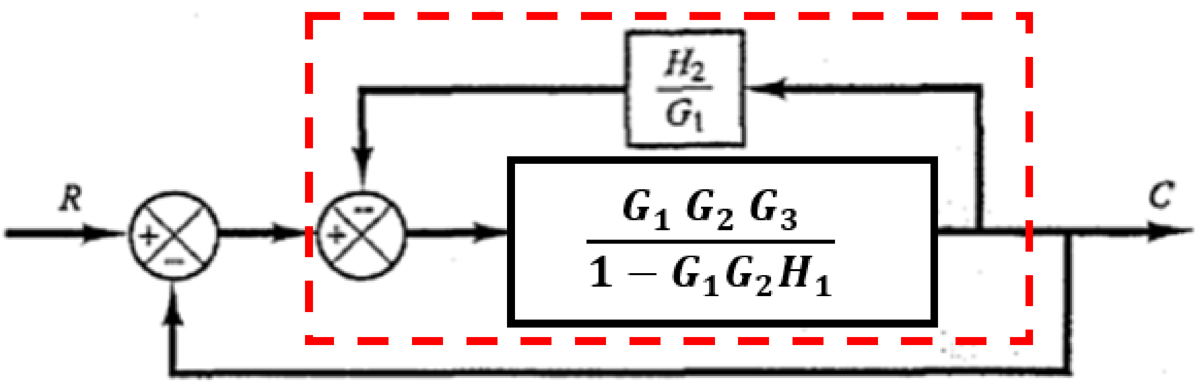


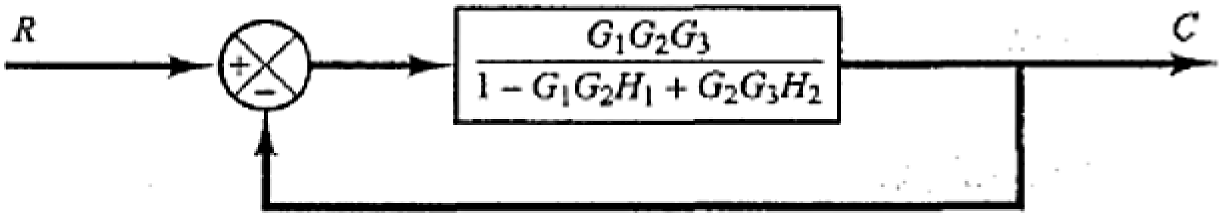
Step 4: Use **Rule 1** for blocks $\frac{G_1 G_2}{1-G_1 G_2 H_1}$ and G_3 which are connected in series to combine them into one block $\frac{G_1 G_2 G_3}{1-G_1 G_2 H_1}$. The modified block diagram is shown in the following figure.



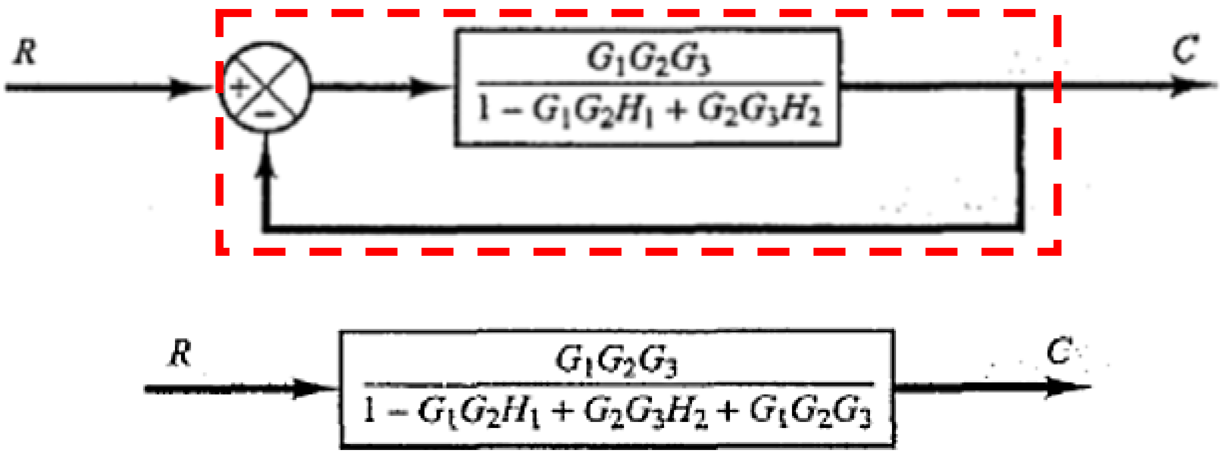


Step 5: Use **Rule 3** for blocks $\frac{G_1 G_2 G_3}{1-G_1 G_2 H_1}$ and $\frac{H_2}{G_1}$ which are connected in negative feedback loop to combine them into one block. The modified block diagram is shown in the following figure.





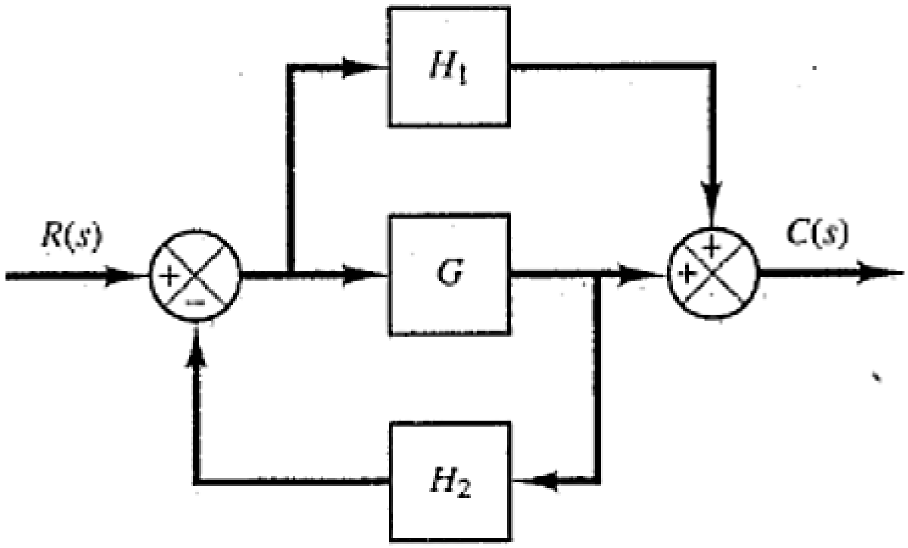
Step 6: Use **Rule 3** for blocks $\frac{G_1 G_2\, G_3}{1 - G_1 G_2\, H_{1+}\, G_2 G_3\, H_2}$ and $H\ = 1$ which are connected in negative feedback loop to combine them into one block. The modified block diagram is shown in the following figure.



Therefore, the **transfer function** of the system is:

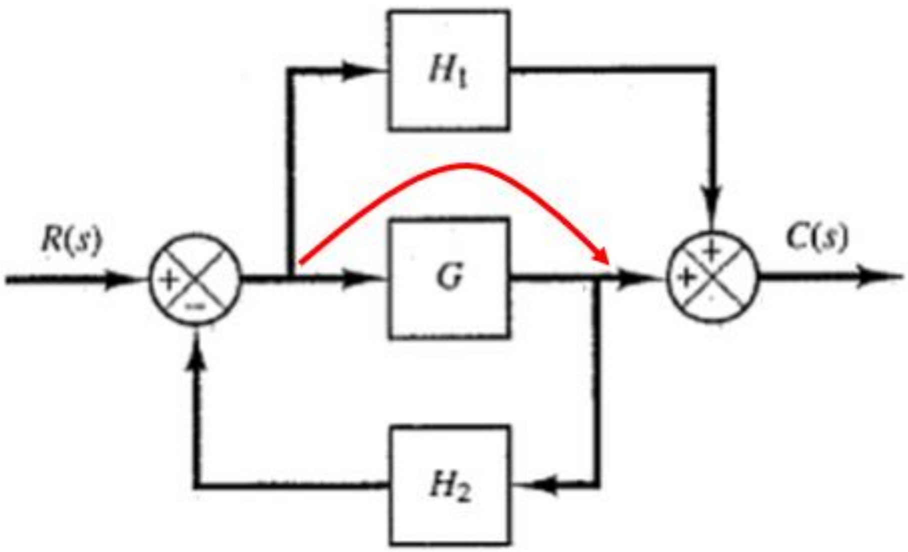
$$\frac{C}{R} = \frac{G_1 G_2\, G_3}{1 - \ G_1 G_2\, H_1 +\ G_2 G_3\, H_2 +\ G_1 G_2\, G_3}$$

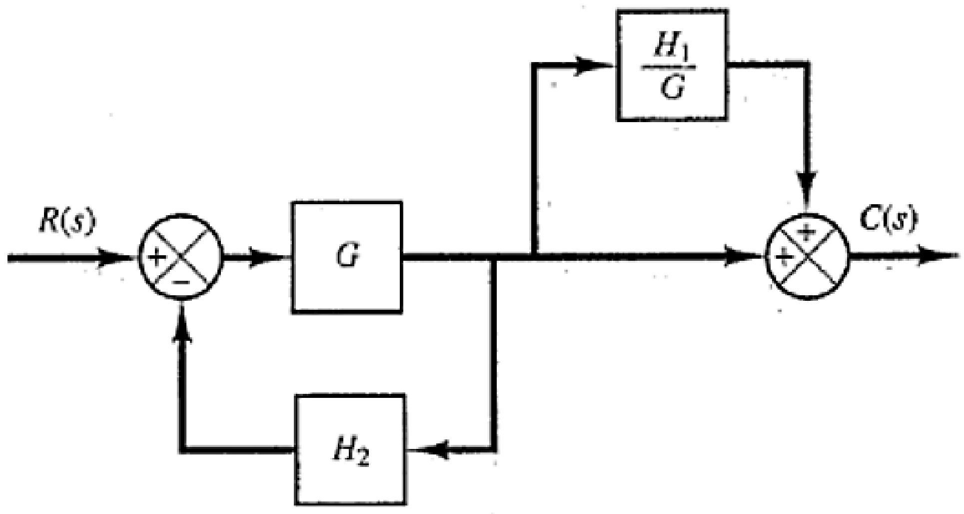
Example 2: Using the block diagram reduction technique, find the transfer function for the block diagram shown in figure below.



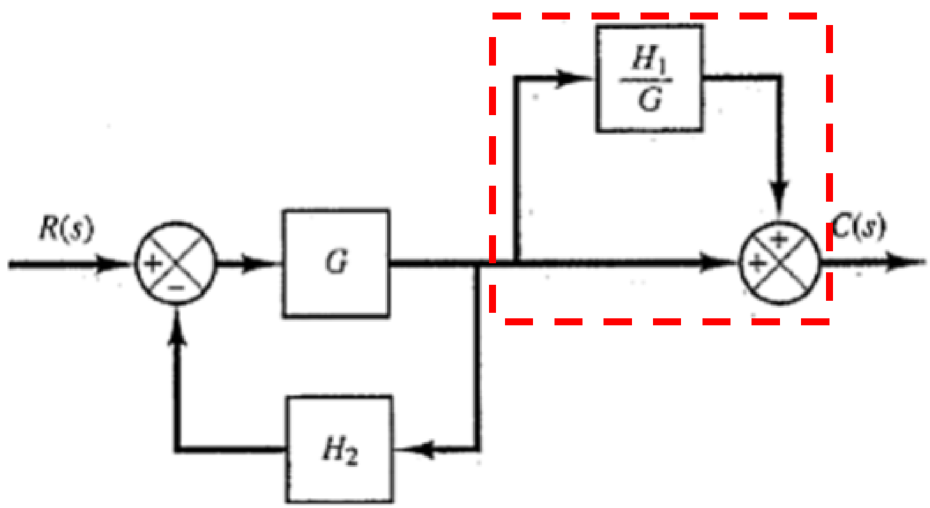
Solution:

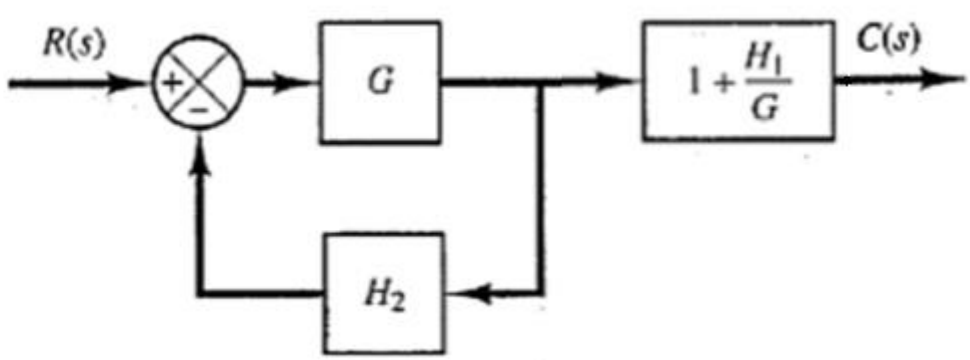
Step 1: There is difficulty with take-off point before G while simplifying. Thus, we use **Rule 4** to shift it towards right (after G).



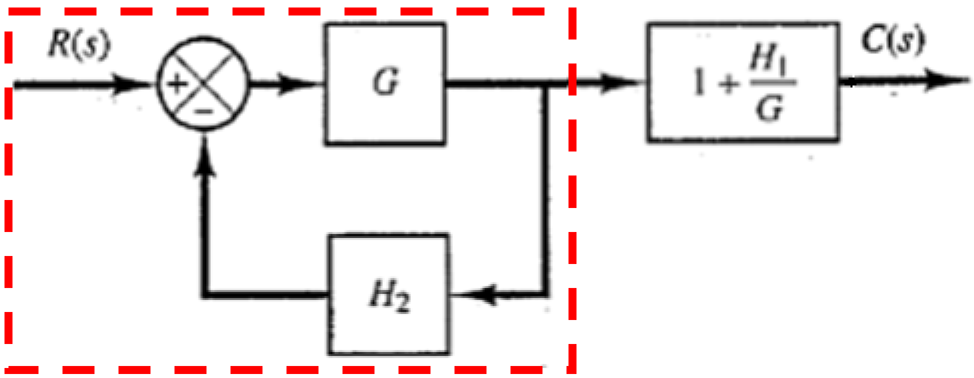


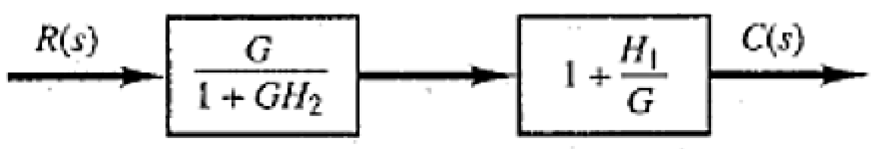
Step 2: Use **Rule 2** for blocks $\frac{H_1}{G}$ and $G(s) = 1$ which are connected in parallel to combine them into one block $1 + \frac{H_1}{G}$. The modified block diagram is shown in the following figure.



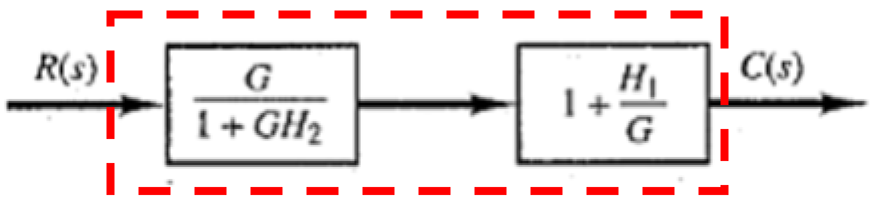


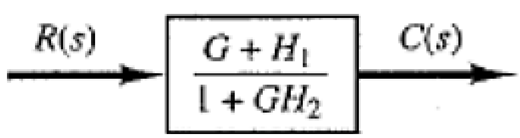
Step 3: Use **Rule 3** for blocks G and H_2 which are connected in negative feedback loop to combine them into one block. The modified block diagram is shown in the following figure.





Step 4: Use **Rule 1** for blocks $\frac{G}{1+G\,H_2}$ and $1+\frac{H_1}{G}$ which are connected in series to combine them into one block $\frac{G+H_1}{1+GH_2}$. The modified block diagram is shown in the following figure.

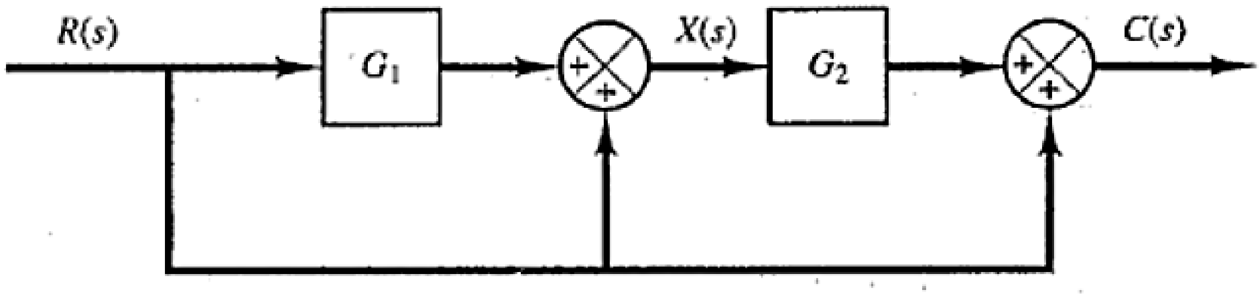




Therefore, the **transfer function** of the system is:

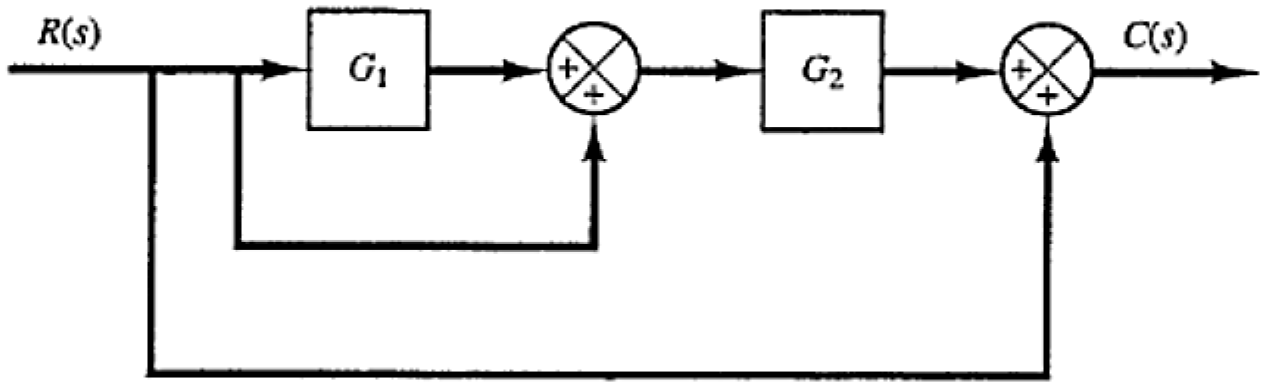
$$\frac{C(s)}{R(s)} = \frac{G+H_1}{1+GH_2}$$

Example 3: Reduce the block diagram in figure below to its simplest possible form and obtain its closed loop transfer function.

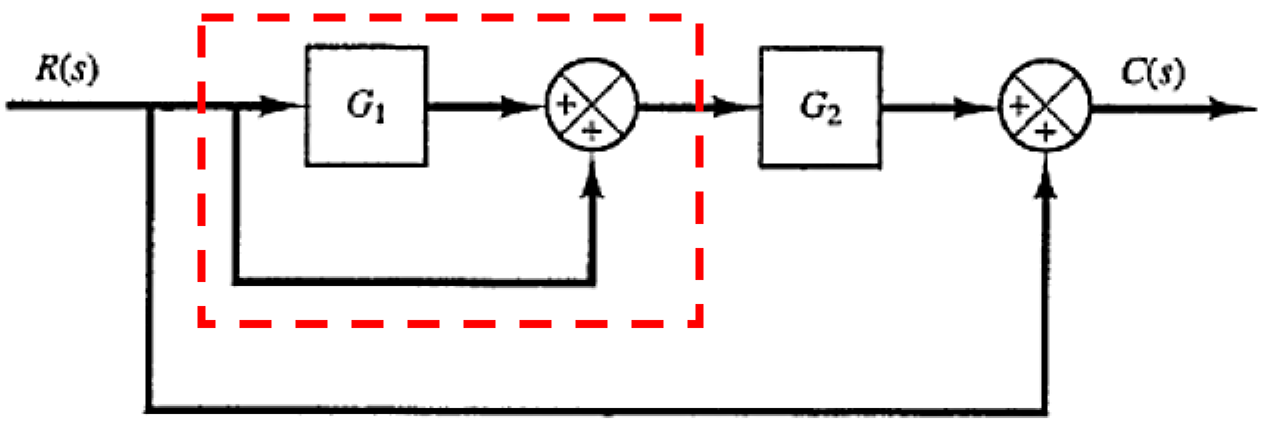


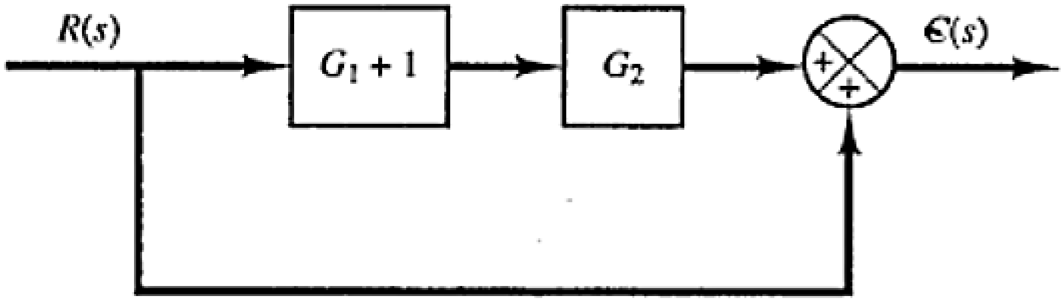
Solution:

Step 1: Separating the feed forward paths, we obtain figure below:

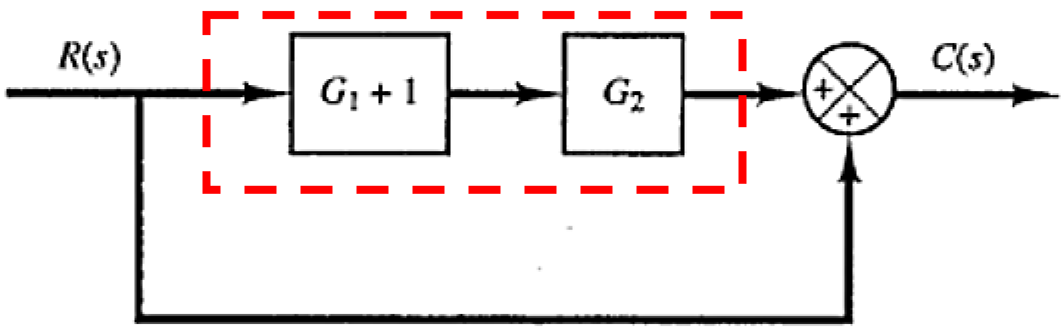


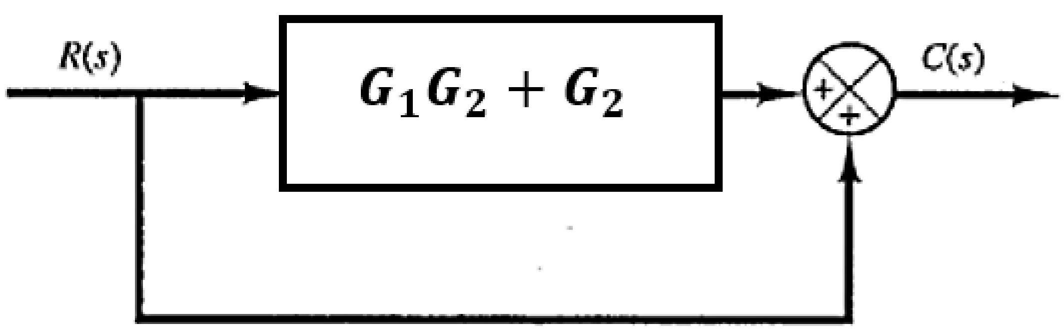
Step 2: Use **Rule 2** for blocks G_1 and $H(s) = 1$ which are connected in parallel to combine them into one block $G_1 + 1$. The modified block diagram is shown in the following figure.



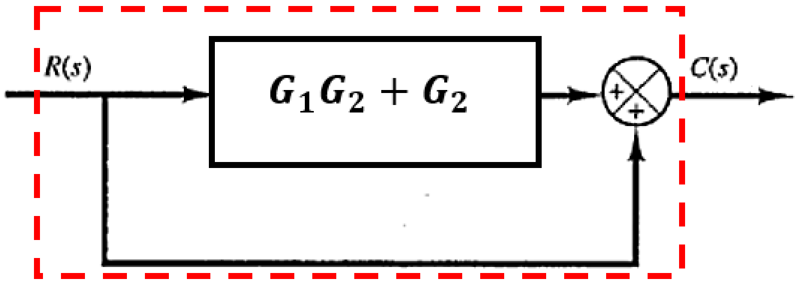


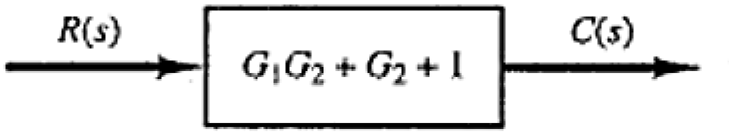
Step 3: Use **Rule 1** for blocks $G_1 + 1$ and G_2 which are connected in series to combine them into one block $G_1 G_2 + G_2$. The modified block diagram is shown in the following figure.





Step 4: Use **Rule 2** for blocks $G_1 G_2 + G_2$ and $G(s) = 1$ which are connected in parallel to combine them into one block $G_1 G_2 + G_2 + 1$. The modified block diagram is shown in the following figure.

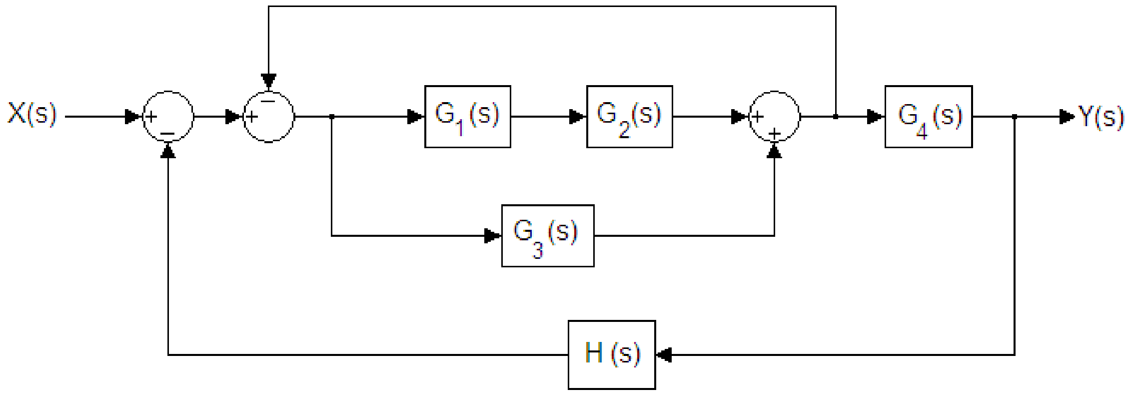




Therefore, the **transfer function** of the system is:

$$\frac{C(s)}{R(s)} = G_1G_2 + G_2 + 1$$

Homework: Simplify the block diagram shown below into a single block and determine the overall transfer function.



ANS.

$$\frac{Y(s)}{X(x)} = \frac{\dfrac{G_1(s)G_2(s)G_4(s) + G_3(s)G_4(s)}{1 + G_1(s)G_2(s) + G_3(s)}}{1 + \dfrac{G_1(s)G_2(s)G_4(s) + G_3(s)G_4(s)}{1 + G_1(s)G_2(s) + G_3(s)} H(s)}$$

$$= \frac{G_1(s)G_2(s)G_4(s) + G_3(s)G_4(s)}{1 + G_1(s)G_2(s) + G_3(s) + G_1(s)G_2(s)G_4(s)H(s) + G_3(s)G_4(s)H(s)}$$